Use of Fuzzy Logic in Determining Quality of Water

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Abstract

There is no hard and fast rule for defining an environmental factor, with respect to a person. For example, a person might find a certain day 'cold' while someone else might find the same day 'mild'. Does that mean one of them is wrong?

The above example illustrates that language can tend to be imprecise and a simple concept like a cold day does not have well defined boundaries as such. The use of Fuzzy Logic helps us to process such data, by extending classical set theory to handle partial membership. Classical set theory deals with crisp sets, i.e. members are either in or out. With fuzzy logic we can deal with partial membership too.

Fuzzy sets can be applied to environmental science. Environmental issues deal constantly with fuzzy concepts such as hazardous, acceptable, safe, etc. It is difficult to have an absolute distinction between such assessments.

Fuzzy concepts come largely from the field of soft computing, and have links to many earlier influences. Working with fuzzy concepts requires skills like constructing fuzzy sets, performing logical operations and also arithmetic operations on these sets.

Traditional classification methods of the water quality parameters use crisp sets, and the concentration values which are close or far from the limits are considered in same classes. Moreover, usually, several parameters are considered in quality determination; therefore, differences of the classes of the parameters may be vagueness, especially, in public consideration.

In this paper, we consider how fuzzy logic and fuzzy arithmetic apply to risk assessment and environmental policy and, specifically, how these concepts apply to a case-study assessment of water quality in India's Mithi. River. Our goal is to consider whether and to what extent this approach can be applied more broadly for environmental assessments.

Index terms: Fuzzy logic, Fuzzy sets, quality of water, mithi river.

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1. Introduction

Water quality observations have little significance by themselves. A pollution parameter which has a specific value is usually meaningful only in the context of knowledge of natural background levels and regulations. Conventional water quality regulations contain quality classes which use crisp sets, and the limits between different classes have inherent imprecision. The methods which contain upper and lower limits have two ambiguities. Firstly, the traditional water quality evaluation methods use discrete form. This classification technique may cause a rough and imprecise approach for data, as in this approach, a parameter being close or far from the limit has equal importance for evaluation of concentration.

Secondly, each quality parameter may belong to one of four classes. That is, all of the parameters may not be included in a single class. These established various quality classes in one sampling location may constitute confusion (ambiguity) for quality definition of that sampling location.

Fuzzy logic can be viewed as a language that allows one to translate sophisticated statements from natural language into a mathematical formalism. Fuzzy logic can deal with highly variable, linguistic, vague and uncertain data or knowledge and, therefore, has the ability to allow for a logical, reliable and transparent information stream from data collection to data usage in environmental applications. The concept of fuzzy logic, which is a mathematical discipline based on fuzzy set theory and express multiple levels process among [0, 1] instead of two levels in classical mathematic (0, 1).

2. Fuzzy sets, logic, and arithmetic

Fuzzy concepts come largely from the field of soft computing and have links to many earlier influences. Confronting fuzzy concepts requires three skills: constructing fuzzy sets (those with partial membership) and performing logical operations and arithmetic operations on those sets. We used each of these capabilities to carry out our case study.

2.1 Fuzzy sets

In contrast to classical sets, fuzzy sets include objects with partial membership. Some view a person 45 years of age

as "old", and others view the person as "young". So this person's age has partial membership in both the "old" and the "young" fuzzy sets. The process of defining membership produces a membership function for these fuzzy sets. Figure 1 shows example membership functions in the linguistic sets "cold", "mild", and "hot" for a range of temperatures. In the language of fuzzy sets, this figure represents membership functions μA, which express the degree of membership of elements x (temperatures) in the set A, where A = cold, mild, or hot. The function μA is a set of ordered pairs in which the first element of the ordered pair is from the set x of temperatures and the second element is from the interval [0, 1] and expresses degree of membership in A. Here, 0 represents non- membership, 1 represents complete membership, and values in between represent intermediate degrees of membership. Membership in a fuzzy set is determined either by observation or by eliciting characterizations from experts or users. In contrast to probability density functions, fuzzy membership functions express the possibility of an outcome rather than the likelihood of an outcome. In a probabilistic approach, we model uncertainty by expressing our belief that an event either occurs or does not. But with fuzzy logic, we model uncertainty as the degree of membership in the set that defines an outcome.

2.2 Fuzzy logic:

Fuzzy logic has become a common way of dealing with information in various fields, such as control theory, smart machines, and investment analysis. But fuzzy sets have also been applied to environmental science and policy. Despite the relevance of fuzzy logic and early efforts to promote its use in risk assessment fuzzy logic applications in risk assessments are still rare.
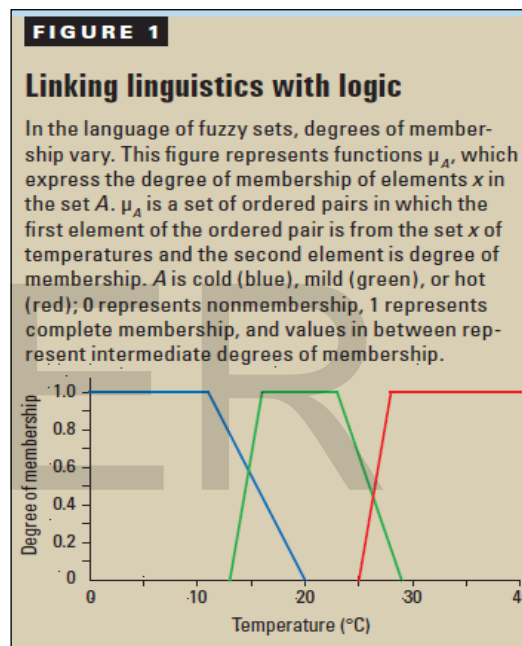
Three basic operations apply to fuzzy sets: negation, intersection, and union. To negate a fuzzy set, simply subtract the membership value in the fuzzy set from 1. For example, in Figure 1, the membership value in "cold" at 5 °C is 1. With negation, the membership value at 5 °C becomes 0. The intersection of two sets is the minimum of the two membership values at each point on the x axis. In Figure 1,the fuzzy set "cold" has a membership value of 0.7 corresponding to x = 14 °C, and the fuzzy set "mild" has a membership value of 0.3 corresponding to x = 14 °C. The intersection has a membership value of 0.3 at x = 14 °C. The union of two sets is the maximum of the two membership values at each point on the x axis. In Figure 1, the union of the sets "cold" and "mild" at x = 14 °C has a membership value of 0.7. In mathematical terms

Negation: $\mu not A(x) = 1 - \mu A(x)$

Intersection: $\mu A \cap B (x) = Min [\mu A(x), \mu B (x)]$

Union: $\mu A \cup B (x) = Max [\mu A(x), \mu B (x)]$

For fuzzy sets, there are extensions of these standard set operations and fuzzy sets operators that have no counterparts in ordinary set theory. For example, families of standard functions, such as the triangular norm (t-norm) for intersection and the t-conorm (or s-norm) for unions, introduce different options for binary mappings to aggregate two membership functions. Operations that are unique to fuzzy sets also include concentration, dilation, normalization, intensification, and fuzzification.

FIGURE 1

Linking linguistics with logic

In the language of fuzzy sets, degrees of membership vary. This figure represents functions μ_A, which express the degree of membership of elements x in the set A. μ_A is a set of ordered pairs in which the first element of the ordered pair is from the set x of temperatures and the second element is degree of membership. A is cold (blue), mild (green), or hot (red); 0 represents nonmembership, 1 represents complete membership, and values in between represent intermediate degrees of membership.

These operators leave classical (crisp) sets unchanged. The concentration operation reduces the membership of marginal elements by squaring the degree of membership of each element in the set. Dilation expands the membership function of the peripheral elements by taking the positive square root of the degree of membership of each set element. Normalization modifies the membership value of all elements by the factor needed to increase the membership status of at least one member to a maximum of 1. Intensification makes a fuzzy set less fuzzy (more defined) by increasing the degree of membership of all set elements that have membership >0.5 by a defined factor and decreasing the degree of membership of all elements with membership <0.5. Fuzzification operates in reverse of intensification and makes the set fuzzier (less defined). Among the important arithmetic operations on fuzzy sets are addition, subtraction, multiplication, division, and degree of match (DM), which we define here and use in the case study.

For example, if we add numbers from set A in the range [1, 5] to numbers from set B in the range [2, 4], we obtain a set of numbers C in the range [3, 9]. Addition of members, x, of set A with membership function μA (x) and members, y, of set B with membership function μB (y) produces elements, z = x + y, of set C that has membership function μC (z):

$$\mu C\ (z) = Min[\mu A(x), \mu B\ (y)]$$

$$sup\ z = x + y$$

Therefore, the degree of membership in C for each pair x, y derived from A and B is calculated as

Min [μA(x), μB (y)].

Then, μC (z) is determined as the maximum (sup) among all combinations x + y that produce a given z value. A similar approach applies to subtraction, multiplication, and division.

On the other hand, the DM operator plays a role in fuzzy-rule-based systems. DM is the measure of overlap in the membership functions of two fuzzy sets. For arbitrary sets A and B:

$$DM(A,B) = \frac{\int \mu_{A \cap B}(X)\,dx}{\int \mu_A(X)\,dx}$$

in which x denotes the values of a parameter, such as dissolved oxygen or fecal coliform (FC) level, and μA∩B (x) is the membership function for the intersection of fuzzy sets A and B.

3. Case Study : Mithi River, Mumbai

The Mithi River (aka Mahim River) is a river in Salsette Island, the island of the city of Mumbai. It is a confluence of tail water discharges of Powai and Vihar lakes. The river is seasonal and rises during the monsoons. The overflowing lakes also contribute to the river flow which is stopped by a dam in other times. During this season the river is a favourite with the anglers who catch large fish that have escaped from the lakes. Fishing is banned there.

The river originates from the overflow of Vihar Lake and also receives the overflows from the Powai Lake about 2 km later. It flows for a total of 15 km before it meets the Arabian Sea at Mahim Creek flowing through residential and industrial complexes of Powai, Saki Naka, Kurla, Kalina, Vakola, Bandra-Kurla complex, Dharavi and Mahim.

It is also less well known that the Mahim bay area, where Mithi River meets Arabian Sea is a nominated bird sanctuary where migratory birds come for nesting. This part is full of mangroves. When the river was not as polluted as it is today, it used to serve as an important storm water drain for Mumbai but as it has been used as a sewer over the years, its importance as a storm water drain has reduced and on the contrary, it poses as a hazard during high tide bringing polluted water into the city.

Water-quality experts have identified five parameters for defining river water quality for bathing—(FC)fecal coliform , dissolved oxygen (DO), biochemical oxygen demand (BOD), pH, and turbidity. Policy makers and the public need information about these values to understand river water quality. But how will they get the information if observed values of these parameters are uncertain because of measurement errors and natural variability? Another approach to communicate water quality is an aggregated water quality index (WQI), which uses the five parameters in a scoring system to express water quality with a number between 0 and 100. If WQI < 20, the water quality is considered to be undesirable or dangerous to bathe in. One problem with this type of highly subjective approach is that the final score fails to communicate the uncertainty in the measurement of the five parameters, the interpretation of an acceptable range for each parameter, and the method used to integrate these dissimilar parameters.

We introduced fuzzy logic to characterize water quality in a way that provides linguistic terms (i.e., highly acceptable, not acceptable) with a certain degree of certainty. The result is an alternative approach with more fidelity to the type of uncertainties involved in this particular problem.

A hierarchical structure for water classification resulting in a set of rules can be constructed. The chemical status of water is judged in the first hierarchical level of knowledge base. The second hierarchical level characterizes bacteriological, chemical and physical status of water to arrive at the ultimate acceptable strategy of water quality for bathing purpose. Following are the sample rules stored at two different hierarchical levels of the knowledge base:

If DO is fair>and BOD is good and pH is every good> Then chemical status of water is good .The rule at the next level could be

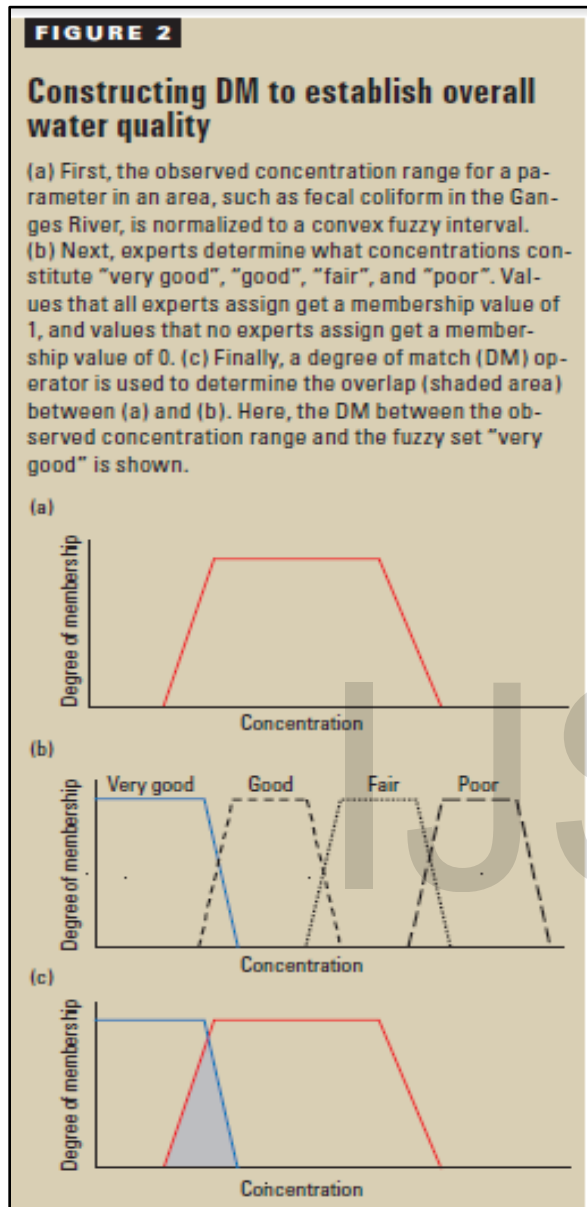If bacteriological status of water is fair and

Chemical status of wafer is good and

Physical status of water is <air

Then water quality for bathing is just acceptable

The degree of match of each classification rule indicates the certainty value of classification.

The greater the degree of match, the greater is the possibility that the object (water) is classified in that class. The rules are processed using conjunction and disjunction operators and as

FIGURE 2

Constructing DM to establish overall water quality

(a) First, the observed concentration range for a parameter in an area, such as fecal coliform in the Ganges River, is normalized to a convex fuzzy interval. (b) Next, experts determine what concentrations constitute "very good", "good", "fair", and "poor". Values that all experts assign get a membership value of 1, and values that no experts assign get a membership value of 0. (c) Finally, a degree of match (DM) operator is used to determine the overlap (shaded area) between (a) and (b). Here, the DM between the observed concentration range and the fuzzy set "very good" is shown.

per the hierarchical structure for fuzzily describing the water quality. The optimal acceptance strategy is usually that for which the degree of assertion is the maximum.

The DM values between observations and fuzzy terms relating to acceptability are used to establish overall water quality. First, we fit the measured parameter to a convex normalized fuzzy interval, as shown in the figure. Here, the membership function represents the range of a given water quality parameter ,such as FC level. Next, we use assessments from experts to construct a model with fuzzy sets that will classify a specific factor, such as FC level, as "very good", "good", "fair", or "poor" . Values to which all experts assign the same term are given a membership value of 1, and values to which no expert assigns that term are given a membership value of 0. We derive values of the membership functions in between by connecting the 0 and 1 membership values with a continuous straight line. Lastly, we use the DM operator to determine the DM between the convex normalized fuzzy set describing observed parameter ranges and the fuzzy sets describing the experts' quality classification ranges, as shown in .Table 1 (on the next page) provides the results of this process and the DM values at Mahim Bay and Bandra Link road for each linguistic class. The numerator for each DM is derived from the fuzzy interval corresponding to the experts' water quality classification. Fuzzy-rule-based system. We construct rules that classify water on the basis of DM quality parameters. Bacteriological status is linked to FC levels and physical status is linked to turbidity, but biochemical status is linked to three secondary attributes—DO, BOD, and pH. At each stage of this hierarchical structure, we apply the experts' acceptability rules to classify water quality and obtain a degree of certainty about the classification.

Sampling Locations	Seasons	pH	DO	FC (µS/cm)	BOD
Mahim Bay	Pre-Monsoon	7.5	0.0	12190.0	139.8
Sion-Bandra Link Road	Pre-Monsoon	7.3	0.2	5294.3	67.4

Table 1: Degree of Match of Field Data with the Fuzzy Values

Sampling locations	Parameters	Linguistic parameters			
		Very good	Good	Fair	Poor
Mahim Bay	Faecal coliform	0.21	0.8	0.36	0
	Dissolved oxygen	0.45	0.16	0	0
	BOD	0.13	0.58	0.4	0.02
	pH	0.41	0.3	0.23	0.1
	Turbidity	0.98	0.03	0	0
Sion-Bandra link road	Faecal coliform	0	0	0.01	0.98
	Dissolved oxygen	0.16	0	0	0
	BOD	0.5	0.33	0.12	0
	pH	0.68	0.47	0.29	0.06
	Turbidity	0.68	0.51	0	0

The river water quality was assessed at various points such as CST Bridge, vakola nalla and Mahim Bridge to have broad idea about river water quality. However, for this study, data of two locations were selected. The first one at Mahim Bay is the outlet of Mithi river where it flows out in the sea. This location witnesses higher dilution due to readily available sea water. On the other hand, water quality below the Sion Bandra link road is highly influenced by the waste water discharges coming from upper regions. This place, the dilution factor is less compared to Mahim Bay location.
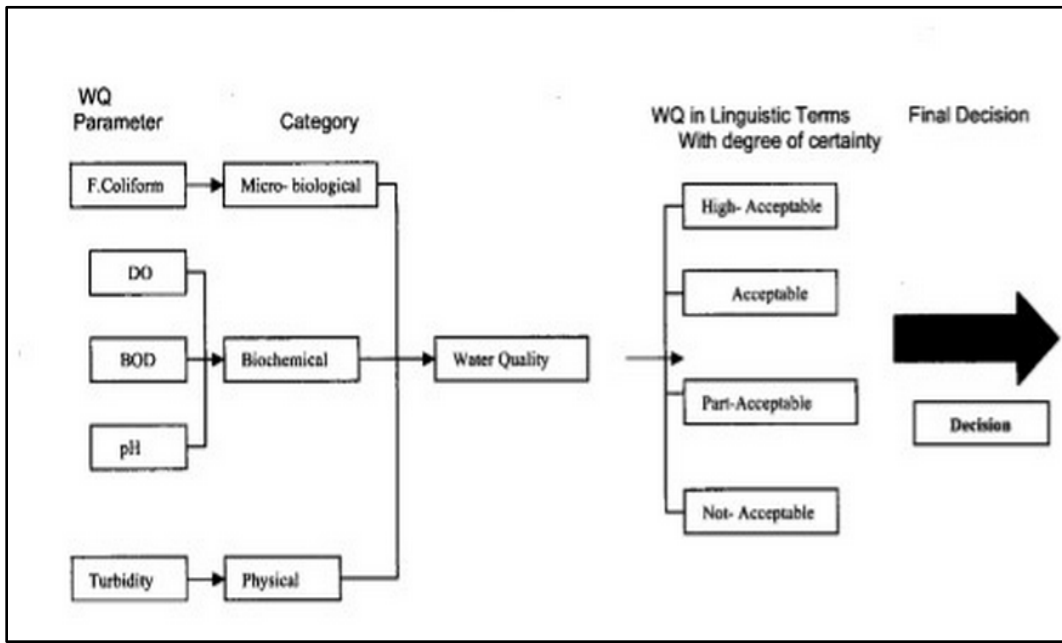
4. Environmental Application:

Over the past few decades, soft computing tools such as fuzzy-logic-based methods, neural networks, and genetic algorithms have had significant and growing impacts. But we have seen only limited use of these methods in environmental fields, such as risk assessment, cost–benefit analysis, and life-cycle impact assessment. Because fuzzy methods offer both new opportunities and unforeseen problems relative to current methods, it is difficult to determine how much impact such methods will have on environmental policies in the coming decades For the types of complex and imprecise problems that arise in environmental policy, the ability to model complex behaviours as a collection of simple if–then rules makes fuzzy logic an appropriate modelling tool. Because fuzzy arithmetic works well for addressing linguistic variables and poorly characterized parameters, fuzzy methods offer the opportunity to evaluate and communicate assessment on the basis of linguistic terms that could possibly match those of decision makers and the public. Moreover, approximate reasoning methods such as fuzzy arithmetic do not require well characterized statistical distributions as inputs. Another key advantage of fuzzy logic in risk assessment is the ability to merge multiple objectives with different values and meanings, for example, combining health objectives with aesthetic objectives. It also provides rules for combining qualitative and quantitative objectives. Perhaps someday a more comprehensive approach that includes exposure surveys, toxicological data, and epidemiological studies coupled with fuzzy modelling will go a long way toward resolving some of the conflict, divisiveness, and controversy in the current regulatory paradigm.

Primary Water Quality Criteria For Class SW-II Waters
(For Bathing, Contact Water Sports and Commercial Fishing)

S.No.	Parameter	Standards	Rationale/Remarks
1.	pH range	6.5-8.5	Range does not cause skin or eye irritation and is also conducive for propagation of aquatic life.
2.	Dissolved Oxygen	4.0 mg/l or 50 percent saturation value, which ever is higher.	Not less than 3.5 mg/l at any time of the year for protection of aquatic lives
3.	Colour and Odour	No noticeable colour or offensive odour.	Specially caused by chemical compounds like creosols, phenols, naptha, pyridine, benzene, toluene etc. causing visible colouration of salt crystal and tainting of fish flesh.
4.	Floating Matters	Nothing obnoxious or detrimental for use purpose.	None in concentration that would impair usages specially assigned to this class.
5.	Turbidity	30 NTU (Nephelo Tur- bidity Unit)	Measured at 0.9 depth..
6.	Fecal Coliform	100/100 ml (MPN)	The average value not exceeding 200/100 ml. in 20 percent of samples in the year and in 3 consecutive samples in monsoon months.
7.	Biochemical Oxygen Demand (BOD) (3 days at 27°C)	3 mg/l	Restricted for bathing (aesthetic quality of water). Also prescribed by IS:2296 1974..

5. Conclusions

Over the past few decades, soft computing tools such as fuzzy-logic based methods, neural networks and genetic algorithms have had significant and growing impacts. When the goal of a certain study is to summarise the observations in an efficient and useful manner, methods using fuzzy logic

should be investigated as an alternative method for addressing uncertain and complex systems.

Fuzzy set theory has revitalised the need for uncertainty analysis in many situations. Fuzzy logic represent a significant change in both the approach and the outcome of environmental evaluations. As of today, risk assessment has an implicit assumption that the probability theory provides the necessary and sufficient tools for dealing with uncertainty and variability. The key advantage of fuzzy methods is how they reflect the human mind in its remarkable ability to store and process information that is consistently imprecise, uncertain and resistant to classification. Fuzzy logic and probability theory are complementary and not competitive.

In the world of soft computing, fuzzy logic has often been the basis of smart machines. However, more efforts and further case studies are required to establish the usefulness of fuzzy logic in risk assessment.

The question that we pose at the end of our study here is, could we, someday, adapt to a system that relaxes the 'crisp lines' and sharp demarcations, to fuzzy gradations? Could we see a day, where a comprehensive approach to environmental issues that includes toxicological data coupled with fuzzy modelling is used to carry out surveys and find solutions to our problems? Maybe we will have to wait and watch to find our answers.

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